Project report on

Synchrotron emission by charged particles in a non-uniform magnetic field

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Contents

Title	Page
1. Introduction	1
1.1 Detection of astrophysical photons	3
1.1.1 Gamma ray detection	3
1.1.2 X-ray detection	3
1.1.3 Visible radiation detection	3
1.1.4 Infrared detection	4
1.1.5 Radio detection	4
2. Synchrotron Emission	4
2.1 Constant magnetic field	4
2.1.1 Total emitted power	6
2.1.2 Power per unit frequency of an emitted radiation	6 8
2.2 Non-Uniform magnetic field	9
2.2.1 For the whole medium	9
2.2.2 Special case of magnetic field distribution	11
2.2.3 For an electron distribution	12
3. Discussion and conclusions	13
4. Future Outlook	14
Acknowledgement	15
References	15

1. Introduction

Our universe contains many mysteries that humans haven't solved yet. Every time we get a little closer to understanding our universe little bit better than before, our universe throws more questions and mysteries at us for us to solve. One of such mysteries is cosmic rays.

Cosmic rays are very high energy radiation which mainly originates from outside our solar system or even milky way galaxy. The range of energies encompassed by cosmic rays is truly enormous, starting at about 10^7 eV and reaching 10^{20} eV for the most energetic cosmic ray ever detected. We still don't understand everything about these cosmic rays.

One of the ways by which we characterise and study properties of cosmic rays is with the help of the so-called cosmic ray spectrum. When we plot the range of energies against the number of cosmic rays detected at each energy(Flux) we generate a cosmic ray spectrum.

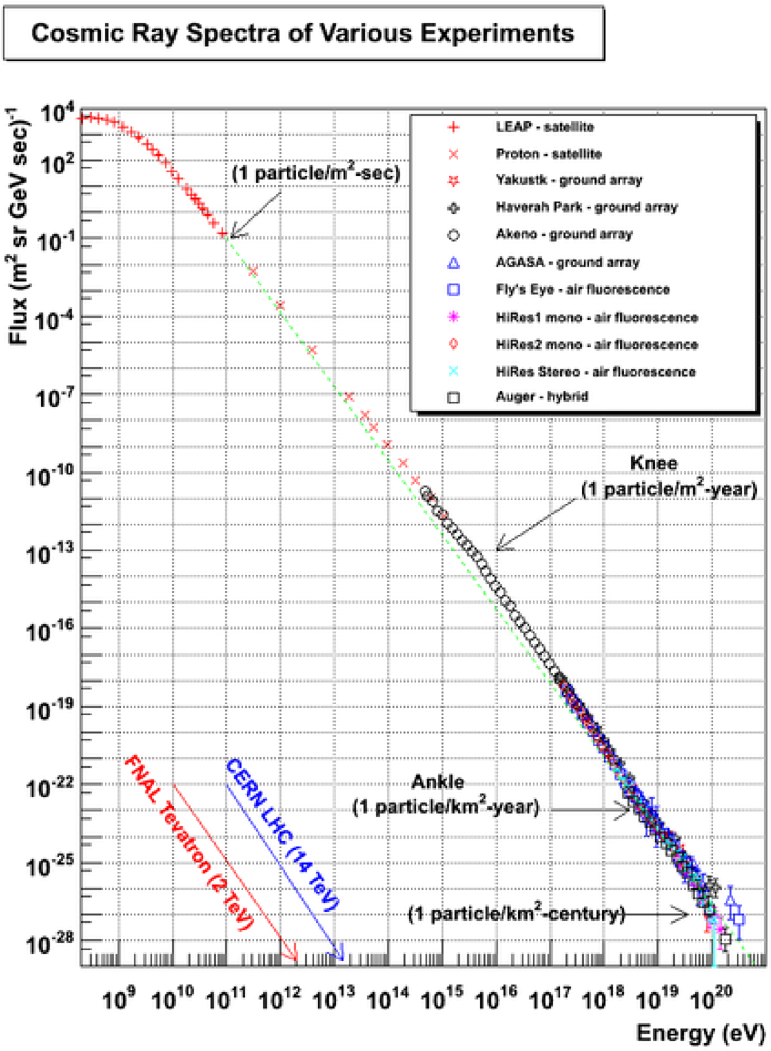

Figure 1: The Cosmic ray spectrum. Credit-http://www.physics.utah.edu/~whanlon/spectrum.html

Looking at Figure 1, we can see that the number of cosmic rays drops off dramatically as we go to higher energies. The origin of these changes in the steepness of the spectrum is still the subject of intense study, but it is assumed that they distinguish between populations of cosmic rays originating via different mechanisms.

Study of cosmic rays has been a challenging tasks for scientists for decades. Unlike radiation, cosmic rays contains very energetic particles such as protons etc. and these particles are very interactive with their surroundings unlike photons. A cosmic ray beam containing such energetic particles can easily change its path by interacting with magnetic or electric fields. Due to this we will get wrong data regarding the actual position of the cosmic ray source. Luckily, these cosmic rays emit radiations while they interact with their surroundings via non-thermal radiative processes and by studying those emitted radiations that we receive on earth we can learn about the original cosmic ray beam. Therefore, studying those radiative processes is a very important part of high energy astrophysics.

The difference between thermal and non-thermal radiative processes is that in the case of a non-thermal radiative process, the characteristics of the emitted radiation do not depend on the temperature of the source. Some of the non-thermal radiative processes is mentioned below-

- The 21cm radiation due to the Hydrogen spin-flip transition
- Gamma rays due to nuclear reactions
- Synchrotron radiation
- Radiation due to Bremsstrahlung
- Inverse Compton process

One thing to notice is the fact that it is not necessary that all x-rays and gamma rays that we receive are due to the non-radiative processes. The black body spectrum extends to infinity in terms of frequency, but the energy density becomes very low there. If an object is hot enough, thermal X-rays are still possible. So the deciding factor between thermal and non-thermal is not so much the wavelength of the photon, but its origin.

Therefore, along with the quantitative properties of the high energy radiation that we receive on earth, it is very important to study the qualitative properties of those radiations such as the origin and the process by which that radiation is emitted.

In this project, I have worked on understanding one of such qualitative aspects of high energy radiation. Understanding the origin and predicting few properties such as the power of the radiation by sources such as supernova remnants, pulsars, nebulas etc. is the main motivation behind this project. Synchrotron is one of the main and most common processes by which high energy radiations are emitted by it's source. Examples of such sources include SNR, AGN, PWN etc. In this project, I have studied the synchrotron emission using basic electromagnetic approach. First I have developed the theory for finding the power emitted by an electron moving in a magnetic field which has both magnitude and direction constant. After this I have extended this theory to include such magnetic fields which have constant magnitude but random directions and finally I have calculated the power per unit frequency of the radiation emitted by electrons moving in a magnetic field which has both non-uniform magnitude and random direction.

1.1. Detection of astrophysical photons

Any theoretical prediction should be accompanied by experimental data which can back that theory and confirm if the calculation based on that theory is right or wrong. There are many ways to detect and measure the amount of radiation that are being emitted by different celestial sources.

1.1.1. Gamma ray detection

On land we can't directly measure the gamma rays from the sources as the gamma rays that are directed towards the earth interacts which the earth's atmosphere and they interact with atoms in the atmosphere and produce ions by dislodging electrons from the atoms. These secondary ions and electrons rain down on earth. These secondary ions and particles can travel faster than the speed of light in that medium. Charged particles moving through the atmosphere with a velocity larger than the local speed of light (the vacuum speed of light divided by the refractive index of the air) emit Cherenkov light. The land based gamma ray detectors detect these secondary ions and electrons from earth's atmosphere to study the initial gamma rays that are being bombarded on earth by faraway sources.

One of such detectors is in India and is called TACTIC(TeV Atmospheric Cherenkov Telescope with Imaging Camera). It is equipped with a light collector and a medium resolution imaging camera of 349 pixels and has been in operation at Mt. Abu, India, since 2001(Yadav, Bhattacharyya, Bhatt, 2007).

1.1.2. X-ray detection

Similar to the gamma rays X-ray photons are absorbed by the earth's atmosphere when it interacts with the atoms of our atmosphere. The energy of the X-ray goes into tearing one of the electrons away from its orbit around the nucleus of a nitrogen or an oxygen atom. This process is called photo-electric absorption.

Due to this reason, most of the X-ray telescopes are space based. One of such telescopes is called ART-P X-ray telescope which has an energy range of 4 to 60 KeV.

1.1.3. Visible radiation detection

Visible radiation (VIS) refer to the wavelength range between 400 nm and 800 nm, which can be perceived by the human eye. One of the challenges with visible radiation is that it gets scattered due to earth's atmosphere therefore while building a big optical telescope we choose place which is high on altitude so that the thickness of atmosphere above our field of vision gets low.

Gran Telescopio Canarias is one of the famous visible band reflecting type telescope located in Spain.

1.1.4. Infrared detection

Photons in energy range 0.12-50 keV constitute the infrared spectrum. Ground-based telescopes have limitations because water vapour in the Earth's atmosphere absorbs infrared radiation. Ground-based infrared telescopes tend to be placed on high mountains and in very dry climates to improve visibility. So most infrared telescopes are also space based.

Wide-field Infrared Survey Explorer (WISE) is an NASA infrared-wavelength astronomical space telescope launched in December 2009.

1.1.5. Radio detection

Radio waves are a type of electromagnetic radiation with wavelengths in the electromagnetic spectrum longer than infrared light. Radio waves have frequencies as high as 300 gigahertz (GHz) to as low as 30 hertz (Hz).

Arecibo radio telescope located at Puerto Rico is one of the many ground based radio telescopes present.

2. Synchrotron Emission

Synchrotron emission is one of the non-thermal radiative processes responsible for high energy radiations that are bombarded on our earth’s atmosphere by celestial sources. Synchrotron process is similar to the cyclotron process, except in synchrotron process the charged particles are moving at a relativistic speed whereas in cyclotron process the charged particles are moving in non-relativistic speeds. Therefore, we have to take relativistic effect in account when dealing with synchrotron emissions. The radiation emitted by synchrotron process is also subjected to beaming effect and so that is also needed to take into account when we calculate the spectrum of that radiation.

In this section, we will calculate and find out how does the power of synchrotron radiation emitted by an electron when travelling in uniform and non-uniform magnetic fields, changes.

2.1. Constant magnetic field

Let we have an electron which is moving in a constant magnetic field having magnetic field vector $\boldsymbol{B}$ and let we don’t have any external electric field in the region. The electron will feel the Lorentz force due to the magnetic field in that region,

$$\frac{d(\gamma m\boldsymbol{v})}{dt} = \frac{q}{c}(\boldsymbol{v} \times \boldsymbol{B}) \tag{1}$$

$$\frac{d(\gamma mc^2)}{dt} = q(\boldsymbol{v}.\boldsymbol{E}) \tag{2}$$

As the electric field is zero.

Equation 2 suggests that γ=Constant or $|\boldsymbol{v}|$=Constant. Using this in (1),

$$m\gamma\frac{d\boldsymbol{v}}{dt} = \frac{q}{c}(\boldsymbol{v} \times \mathbf{B}) \tag{3}$$

Separating the velocity components along the field $v_{\parallel}$ and in a plane normal to the field $v_{\perp}$ we have

$$\frac{d\boldsymbol{v}_{\parallel}}{dt} = 0 \tag{4}$$

$$\frac{d\boldsymbol{v}_{\perp}}{dt} = \frac{q}{\gamma mc}(\boldsymbol{v}_{\perp} \times B) \tag{5}$$

From here we can see that the solution to this equation is a circular motion of the projected motion on the normal plane. The combination of this circular motion and the uniform motion along the field is a helical motion of the particle. The frequency of the rotation or gyration can be calculated as,

$$m\omega^2 r = \frac{q}{c}vB \tag{6}$$

$$\omega_B = \frac{qB}{\gamma mc} \tag{7}$$

Here B=Magnitude of vector **B.**

From 4 we can see that,

$$a_{\parallel} = 0 \tag{8}$$

From 5 we can see that,

$$a_{\perp} = w_B v_{\perp} = \frac{qvbv_{\perp}}{\gamma mc} \tag{9}$$

2.1.1. Total emitted power

The total power emitted by radiation is given by,(Rybicki and Lightman, 2004)

$$P = \frac{2q^2}{3c^3}\gamma^4(a_\perp^2 + \gamma^2 a_\parallel^2) \tag{10}$$

From 10, 8 and 9

$$P = \frac{2q^2}{3m^3}\gamma^4 \frac{q^2B^2}{\gamma^2m^2c^2}v_\perp^2 \tag{11}$$

For an isotropic distribution of velocities, it is necessary to average this expression over all angles for a given speed β. Let α be the pitch angle i.e. the angle between field and velocity. Then we would obtain,

$$\langle\beta_\perp^2\rangle = \frac{\beta^2}{4\pi}\int sin^2\alpha \, d\Omega = \frac{\beta^2}{4\pi}\iint sin^3\alpha \, d\alpha d\phi = \frac{2\beta^2}{3} \tag{12}$$

Therefore, the total power for isotropic distribution of velocities would be,

$$P = \frac{4}{3}r_0^2 c\beta^2\gamma^2B^2$$

Where, (Rybicki and Lightman, 2004, chapter 3)

$$r_0 = \frac{e^2}{mc^2}$$

The quantity r_0 gives a measure of the ‘‘size’’ of the point charge, assuming its rest energy mc^2 is purely electromagnetic in origin. For an electron r_0 is called the classical electron radius and has a value $r_0 = 2.82 \times 10^{-13}cm.$

The critical frequency is defined as, (Rybicki and Lightman, 2004)

$$\omega_c = \frac{3}{2}\frac{qB}{mc}\gamma^2 sin\alpha \tag{13}$$

2.1.2. Power per unit frequency of the emitted radiation

The electric field is a function of θ solely through the combination $\gamma\theta$, where θ is a polar angle about the direction of motion. This is a manifestation of the beaming effect. So we can write,

$$E(t) \propto F(\gamma\theta) \tag{14}$$

Where t refer to the time measured in the observers frame. We set the zero of time and the path length s to be when the pulse is centered on the observer. Using the fact that, $\theta \approx {}^{s}/_{a}$ and $t \approx (s/v)(1 - v/c)$ we can write that,

$$\gamma\theta \approx 2\gamma(\gamma^2\omega_B \sin\alpha)t \approx \omega_c t \tag{15}$$

So we can write the time dependence of the electric field as,

$$E(t) \propto g(\omega_c t) \tag{16}$$

The proportionality constant here is not yet known, and it may depend on any physical parameters except time t. This is still sufficcinet for you to derive the general depence of the spectrum on ω. The Fourier transform of the electric field is,

$$\hat{E}(\omega) \propto \int_{-\infty}^{\infty} g(\omega_c t)e^{i\omega t}dt \tag{17}$$

Changing variables of integration to $\xi \equiv \omega_c$ we have,

$$\hat{E}(\omega) \propto \int_{-\infty}^{\infty} g(\xi)e^{i\omega\xi/w_c}d\xi \tag{18}$$

We know that is the pulse repeats on an average time scale T then we can formally write,

$$\frac{dW}{d\omega dt} = T^{-1}\frac{dW}{d\omega} \tag{19}$$

The spectrum $dW/d\omega d\Omega$ is proportional to the square of $\hat{E}(\omega)$. Integrating this over solid angle and dividing by the orbital period, both independent of frequency, and using the above fact then gives us,

$$\int \frac{dW}{dtd\omega d\Omega}d\Omega = \frac{dW}{dtd\omega} = T^{-1}\frac{dW}{d\omega} \equiv P(\omega) = C_1 F(\frac{\omega}{\omega_c}) \tag{20}$$

Where F is a dimensionless function and $C_1$is a constant of proportionality.

We do not know what $\int F(x)dx$ is until we specify $F(x)$. However, we can regard it's non-dimensional value as arbitrary, merely setting a convention for the normalization of $F(x)$. We can still find the dependence of the constant C_1 on all the physical parameters.

From equation 11 and 13,

$$P = \frac{2q^4B^2\gamma^2\beta^2 sin^2\alpha}{3m^2c^3} \tag{21}$$

$$\omega_c = \frac{3}{2}\frac{\gamma^2 qBsin\,\alpha}{mc} \tag{22}$$

So the power per unit frequency emitted by each electron for a constant magnetic field and highly relativistic particle($\beta \approx 1$) is given by,

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3 B sin\,\alpha}{mc^2} F\left(\frac{\omega}{\omega_c}\right) \tag{23}$$

Where, (Rybicki and Lightman, 2004)

$$F(x) = x\int_x^\infty K_{5/3}(t)dt$$

And, K is Bessel's function.

The choice $\sqrt{3}/2\pi$ for the non-dimensional constant has been made to anticipate the convention choice for the normalization of F.

2.2. Non-uniform magnetic field

In the previous section, we considered a constant magnetic field to derive power emitted by an electron in form of radiation in the magnetic field. In reality, having a constant magnetic field in our medium through which the electron accelerates is a very rare situation. The constant magnetic field is an ideal case in our universe. When we look at gamma-ray sources such as nebulas(ex- crab nebula), supernova remnants, faraway galaxies etc. then we find that that the magnetic field present at those sources is far from constant or uniform, so in order to get a better understanding of gamma-ray emission from real sources we need to find the emissivity or power per unit frequency emitted by electrons accelerating in a non uniform magnetic field.

In this section we will find a generalisation of the equation given by equation 23 but we will do that by taking some assumptions and special forms of non-uniform magnetic fields along with necessary approximations.

- Even when the magnetic field is not uniform, if the radius of gyration of an electron at a local point is less than the variation distance for the magnetic field the electron will feel only a single magnetic field strength around a local point, and it will tend to follow a circular or gyrated motion around that point and so all the above calculations done for a uniform magnetic field will be valid at a local point. Thus, the power per unit frequency for an electron at a local point will be the same as above i.e.,

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3 B sin\,\alpha}{mc^2} F\left(\frac{\omega}{\omega_c}\right) \tag{24}$$

2.2.1 For the whole medium

The power emitted per unit frequency by a single electron in a medium with the magnetic field having uniform strength but random direction can be written as(Zirakashvili and Aharonian, 2010)

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3}{mc^2}\int B.P(B)dB\,R\left(\frac{\omega}{\omega_c}\right) \tag{25}$$

Here $R\left(\frac{\omega}{\omega_c}\right)$ is defined as(Crusius, Schlickeiser, 1986)

$$R(x) = \frac{x}{2}\int_0^{\pi} \sin\theta\, d\theta \int_{x/\sin\theta}^{\infty} K_{5/3}(t)dt \tag{26}$$

The function $R(x)$ of the argument $x = \omega/\omega_c$ describes synchrotron radiation of a single electron in a magnetic field with chaotic directions. The function $R(x)$ by definition contains the information about the averaging of the $\sin\alpha$ term of the equation 23. Therefore we don't have to average $\sin\alpha$ term exclusively while finding the power per unit frequency for an electron in magnetic field which has constant magnitude but random direction.

Crusius and Schlinkeiser(1986) derived an exact expression for $R(\omega/\omega_c)$ in terms of Whittaker's function and it is given by,

$$R(x) = \frac{1}{2}\pi x\left[W_{0,\frac{4}{3}}(x)\,W_{0,\frac{1}{3}}(x) - W_{\frac{1}{2},\frac{5}{6}}(x)\,W_{\frac{-1}{2},\frac{5}{6}}(x)\right] \tag{27}$$

Where $W_{\lambda,\mu}(x)$ denotes Whittaker's function(Abramowitz and Stegun, 1970)

With an accuracy of several percent $R(\omega/\omega_c)$ can be presented in a simple analytical form (Zirakashvili and Aharonian 2007),

$$R(x) = \frac{1.81\exp(-x)}{\sqrt{x^{-2/3} + (3.62/\pi)^2}} \tag{28}$$

Where $x = \frac{\omega}{\omega_c}$.

2.2.2 Special case of magnetic field distribution

Now we will consider a special magnetic field probability distribution in order to calculate the power per unit frequency emitted by electrons travelling in that magnetic field.

Many of the supernova remnants that are responsible for synchrotron emission that we observe have turbulent mediums. In a turbulent medium, the magnetic field is not uniform and is described by a probability distribution $P(B)$. The probability distribution found in numerical simulations of the non-resonant streaming instability can be written in a simple analytical form(Zirakashvili and Ptuskin,2008)

$$P(B) = \frac{6B}{B_{rms}^2} \exp\left(-\sqrt{6}B/B_{rms}\right) \tag{29}$$

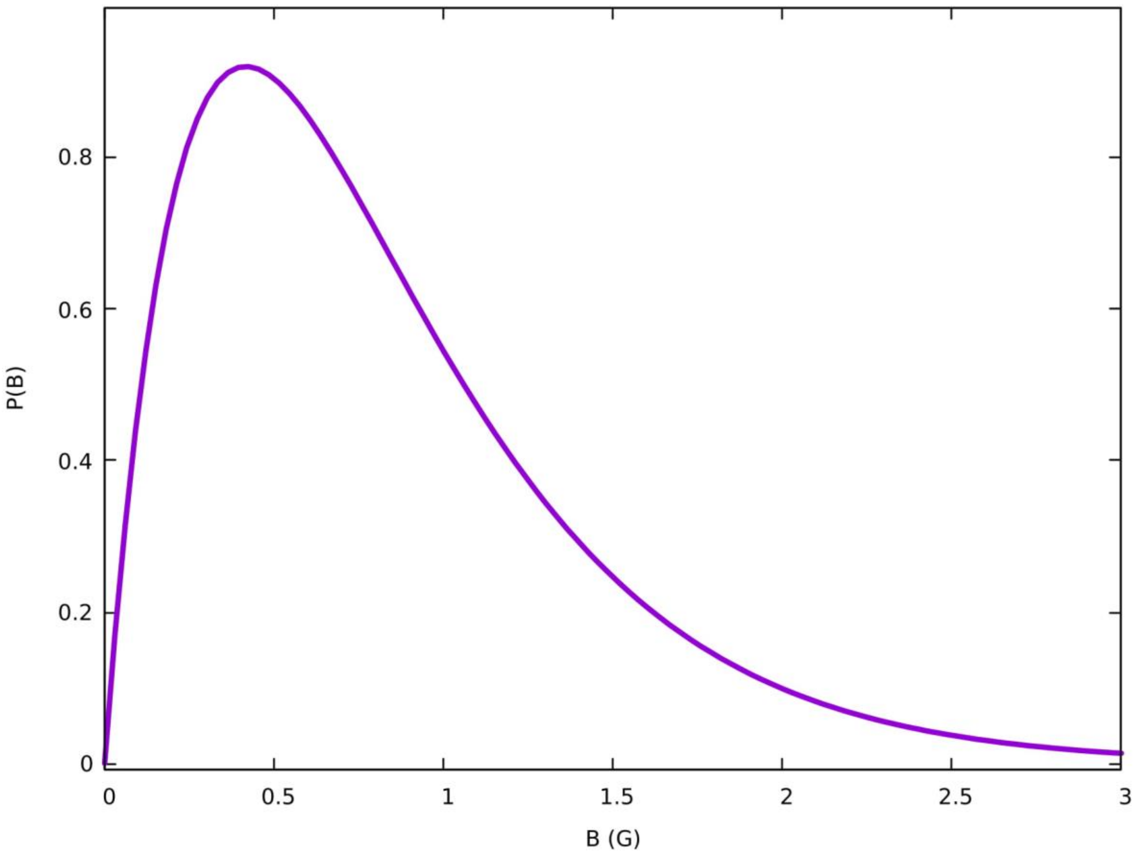

Figure 2: Magnetic field probability density given by the equation 23. Here we have chosen *B_{rms}=1 G.*

We will consider this probability distribution for our further calculations. This magnetic field probability distribution is relevant because in real sources this type of probability distribution is common and this will provide us with an expression which is more close to the real situation than the expression we got by using constant magnetic field.

The synchrotron emission of a single electron in a magnetic field with a probability distribution $P(B)$(as given above) is described by the function $R_1(x)$ such that,

$$R_1(x) = \int B.P(B).R(x)dx \tag{30}$$

Where $x = \frac{\omega}{\omega_c}$.

Using equations 29 in 30 we can see that,

$$R_1(x) = \int B.\left[\frac{6B}{B_{rms}^2} \exp\left(-\sqrt{6}B/B_{rms}\right)\right] dB\ R(x) \tag{31}$$

$$R_1(x) = \int \frac{6B^2}{B_{rms}^2} \exp\left(-\sqrt{6}B/B_{rms}\right) dB\ R(x) \tag{32}$$

Let $y = \frac{B}{B_{rms}}$.

$$R_1(x) = \int 6y^2 \exp\left(-\sqrt{6}y\right).B_{rms} dy\ R(x) \tag{33}$$

$$R_1(x) = \int 6y^2 \exp\left(-\sqrt{6}y\right) dy\ R\left(\frac{x}{y}\right) \tag{34}$$

From 25 and 34 we can see,

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3}{mc^2} \int y^2 R(x/y) \exp\left(-\sqrt{6}y\right) dy \tag{35}$$

There is an analytical approximation for the function $R_1(x)$,(Zirakashvili and Aharonian, 2010)

$$R_1(x) = a x^{1/3} \left(1 + b x^{1/2}\right)^{11/6}. \exp\left(c x^{1/2}\right) \tag{36}$$

Where a=1.50, b=1.53, c=$-96^{1/4}$

Using equation 36 in 35,

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3}{mc^2} \left[a x^{1/3} \left(1 + b x^{1/2}\right)^{11/6}. \exp\left(c x^{1/2}\right)\right] \tag{37}$$

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3}{mc^2} \left\{a \left(\frac{\omega}{\omega_c}\right)^{1/3} \left(1 + b \left(\frac{\omega}{\omega_c}\right)^{1/2}\right)^{11/6}. \exp\left(c \left(\frac{\omega}{\omega_c}\right)^{1/2}\right)\right\} \tag{38}$$

2.2.3 For an electron distribution

Now after calculating everything for a single electron we would like to move further and calculate for an electron distribution.

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3}{mc^2} \int B.P(B) dB \int N(\gamma)\ R\left(\frac{\omega}{\omega_c}\right) d\gamma \tag{39}$$

Where $N(\gamma)$=Electron number density

2.2.4 Special case of electron distribution

We will calculate this for a special case in which we have N_0 number of monoenergetic electrons with energy $\gamma_0 m_e c^2$. So we have,

$$N(\gamma) = N_0\delta(\gamma - \gamma_0) \tag{40}$$

Using equations 38, 39 and 40 we can write,

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3}{mc^2}\int N_0\delta(\gamma - \gamma_0)\left[a\left(\frac{\omega}{\omega_c}\right)^{1/3}\left(1 + b\left(\frac{\omega}{\omega_c}\right)^{1/2}\right)^{11/6}\exp\left(c\left(\frac{\omega}{\omega_c}\right)^{1/2}\right)\right]d\gamma \tag{41}$$

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3}{mc^2}\int N_0\delta(\gamma - \gamma_0)\left[a\left(\frac{2mc\omega}{3\gamma^2 qB}\right)^{1/3}\left(1 + b\left(\frac{2mc\omega}{3\gamma^2 qB}\right)^{1/2}\right)^{11/6}\exp\left(c\left(\frac{2mc\omega}{3\gamma^2 qB}\right)^{1/2}\right)\right]d\gamma$$

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3}{mc^2}N_0\left\{a\left(\frac{2mc\omega}{3\gamma_0^2 qB}\right)^{1/3}\left(1 + b\left(\frac{2mc\omega}{3\gamma_0^2 qB}\right)^{1/2}\right)^{11/6}\exp\left(c\left(\frac{2mc\omega}{3\gamma_0^2 qB}\right)^{1/2}\right)\right\} \tag{42}$$

$$P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3}{mc^2}N_0\left\{a\left(\frac{\omega}{(\omega_c)_{\gamma_0}}\right)^{1/3}\left(1 + b\left(\frac{\omega}{(\omega_c)_{\gamma_0}}\right)^{1/2}\right)^{11/6}\exp\left(c\left(\frac{\omega}{(\omega_c)_{\gamma_0}}\right)^{1/2}\right)\right\} \tag{43}$$

Where a=1.50, b=1.53, c=$-94^{1/4}$, $(\omega_c)_{\gamma_0} = \frac{3qB\gamma_0^2}{2mc}$

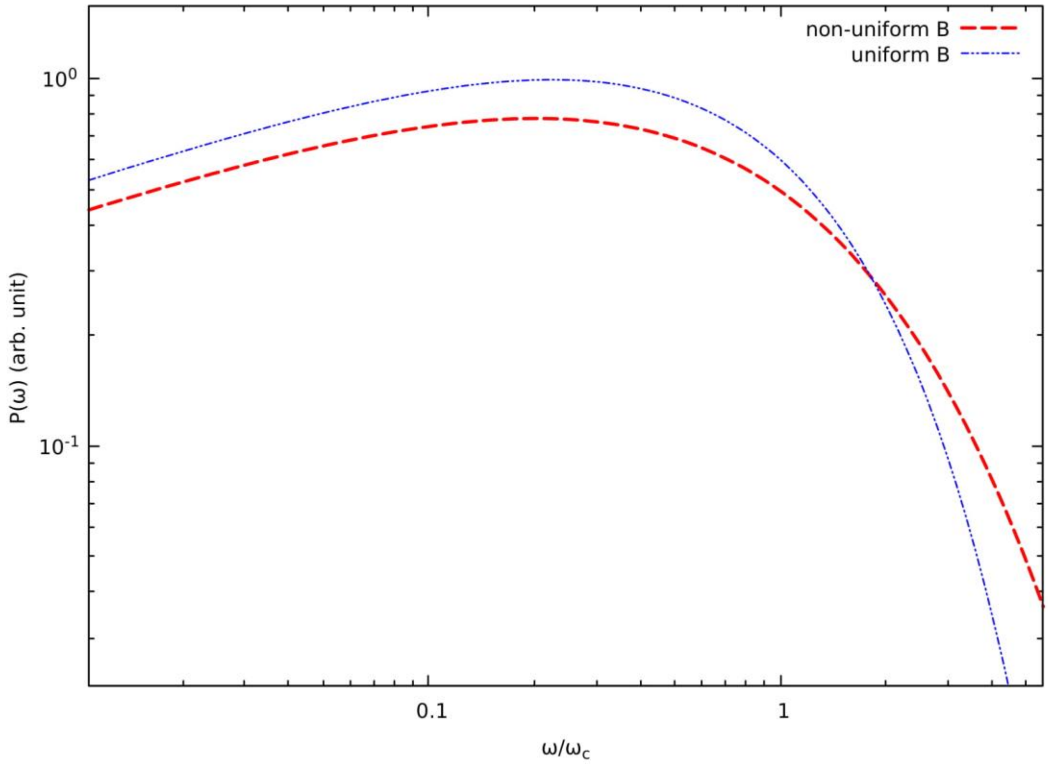

Figure 3: Power emitted per unit frequency in a Synchrotron emission. The dashed line represents a magnetic field with non-uniform magnetic field strength(Magnitude of vector B), and the dotted line represents a magnetic field with uniform magnetic field strength(Magnitude of vector B). Here we took $N_0 = 10^{44}$i.e total number of electrons per unit volume is 10^{44}.

3. Discussion and conclusions

We can see that there are three different functions that are used in the calculations as a replacement of one another. These functions are $F\left(\frac{\omega}{\omega_c}\right)$, $R\left(\frac{\omega}{\omega_c}\right)$ and $R_1\left(\frac{\omega}{\omega_c}\right)$.

The following paragraphs will compile all the information that is already given in the previous sections but in a more accessible and organised manner.

$F\left(\frac{\omega}{\omega_c}\right)$ => This function is derived for the case of constant magnetic field i.e. constant in both magnitude and direction(Rybicki and Lightman). It's a dimensionless function and helps in describing the power per unit frequency for a single electron in a constant magnetic field.

$R\left(\frac{\omega}{\omega_c}\right)$ => This function is derived for the case of a magnetic field which has a constant magnitude but a random direction of orientation(Crusius and Schlickeiser, 1986). This function helps in describing power per unit frequency by a single electron in a large scale random magnetic field.

$R_1\left(\frac{\omega}{\omega_c}\right)$ => This function is derived in case of a magnetic field which has a continuous probability distribution. This function is defined for a particular magnetic probability distribution. For our case it is defined for a magnetic probability distribution defined by equation 29**.** This helps in describing the power per unit frequency by a single electron in a magnetic field which has non uniform magnitude as well as random direction.

Figure 3 shows us that the power spectrum changes on changing the behaviour of the magnetic field from uniform to non-uniform.

We can see in the graph that the peak of the two curves are different or we can say that the peak of the curve for the non-uniform case is shifted towards lower frequency in comparison to the peak of the curve for uniform magnetic field. The peak in the non-uniform case is lower than the uniform case. We can also see that at one particular point the two plots intersect each other. This indicates that there exists a value of ω such that an identical electron in the two different fields will emit radiation with the same power. We can also see that the tail(part of the curve after the intersection point) of the curve for the non-uniform magnetic field falls more slowly than the curve for the uniform field. It indicates that if we have two electrons with same energy such that one is moving in a uniform magnetic field and the other is moving in the non-uniform magnetic field, then we will observe that the electron moving in the non-uniform field will emit more photons as compared to the identical electron moving in a uniform field.

4. Future outlook

This work has much potential to be carried out further, and there are a lot of things that can be added to this work. The method that I have followed while carrying out this project opens many possibilities for one to find many things by just playing with the formulation and mathematics involved.

One of the things that can be improved in this project is the use of a better electron number density. In this project, I have assumed that we have a medium filled with mono-energetic electrons. In real cases, this is not possible. It's an ideal case that we will have all the electrons with the same energy in a medium. In real situations, it has been found out that the power law distribution is very common or at least is a very good approximation to the actual electron distribution. One can improve on this work by considering power law distribution instead of the Dirac delta function that I used. One can also try solving the equations for other exotic electron distributions to find other interesting observations.

The magnetic field probability that I used in this work is a good approximation for many real cases, but one can also improve on that. By using the basic equation, 25 one can find power per unit frequency for any source exclusively. By finding the exact or the most perfect approximation of P(B) for a source and plugging that in 25, one can find an exclusive expression for that particular source which will theoretically fit the data from that source most accurately.

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